PROJECTED WRITTEN NOTES FROM THE VIDEO

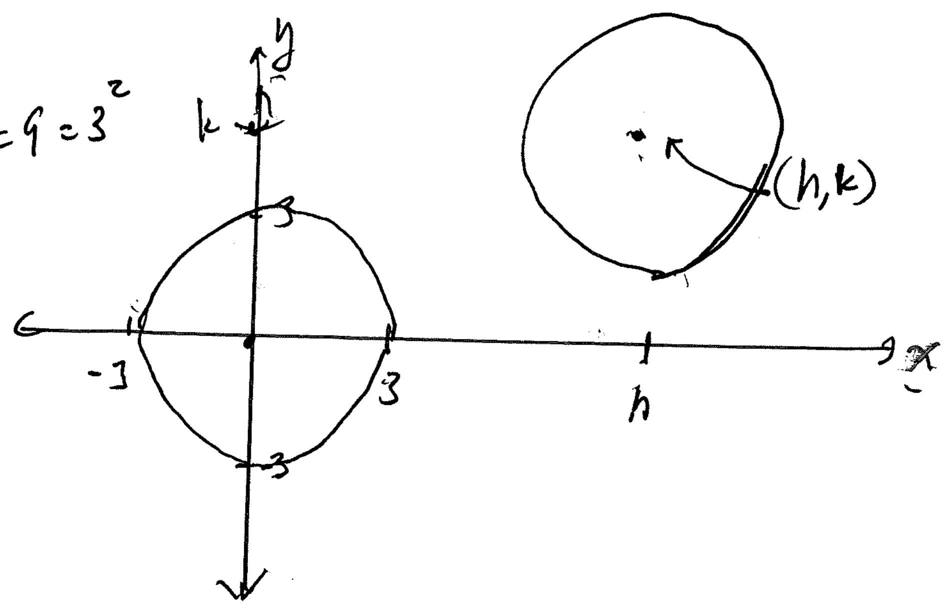
CONIC SECTIONS: PARABOLAS

The Conic Sections: Parabolas, Ellipses, Hyperbolas

A word about shifting

$(x-h)^2+(y-k)^2=9=3^2$

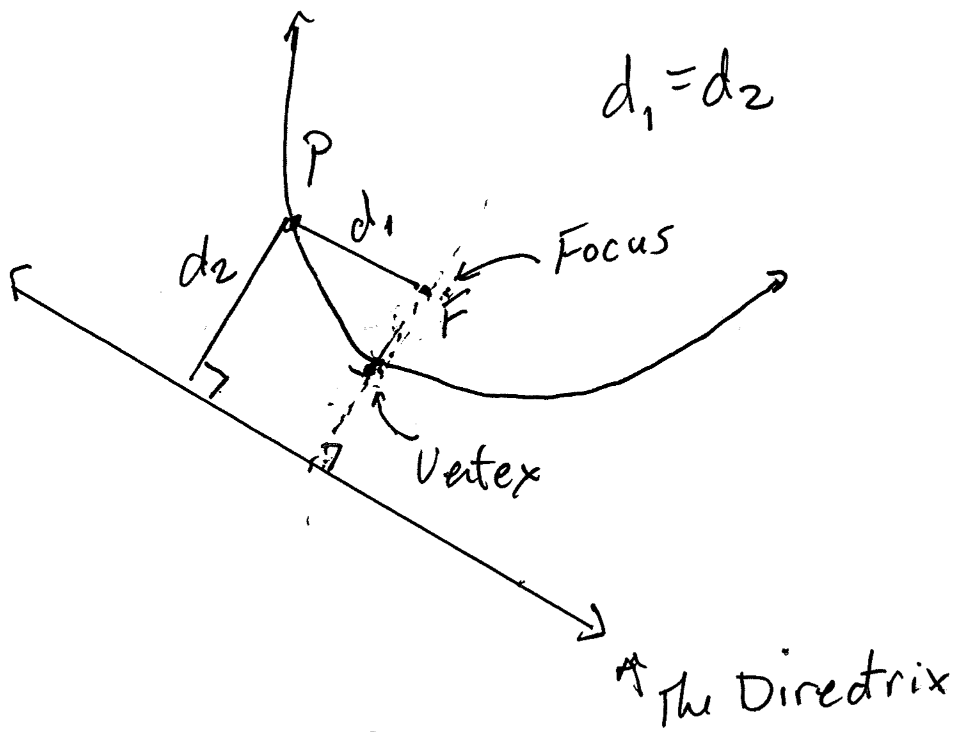
$x^2+y^2=9=3^2$



Replace every occurance of "x" with "$(x-h)$"

Replace every occurance of "y" with "$(y-k)$".

PARABOLAS: $x^2 = 4py$ or $y^2 = 4px$, $p \neq 0$.



In Standard form:

The vertex is $(0,0)$.

$|p|$ = The distance from the Vertex to the Focus.

The Focus is on the Axis of the degree 1 variable

when $x^2=4py$ is the Equation, the Focus is $(0,p)$

The Horizontal Directrix is $y=-p$

when $y^2=4px$ is the Equation the Focus is $(p,0)$

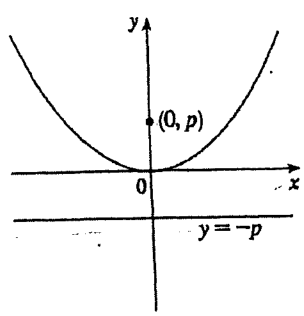
and the Vertical Directrix is $x=-p$.

CONIC SECTIONS

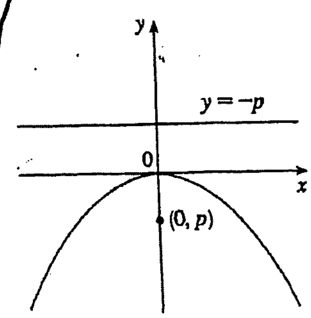
PARABOLAS

1 An equation of the parabola with focus $(0, p)$ and directrix $y = -p$ is

$$x^2 = 4py$$

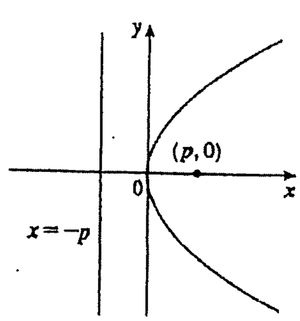


(a) $x^2 = 4py,\ p > 0$

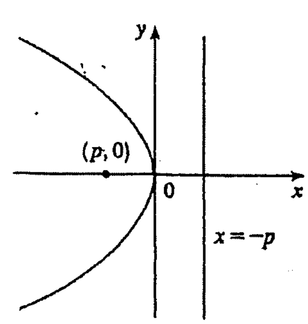


(b) $x^2 = 4py,\ p < 0$

$$y^2 = 4px$$



(c) $y^2 = 4px,\ p > 0$



(d) $y^2 = 4px,\ p < 0$

Problem: For the Parabola $x^2 = 12y$,

Find the Focus F, The vertex V

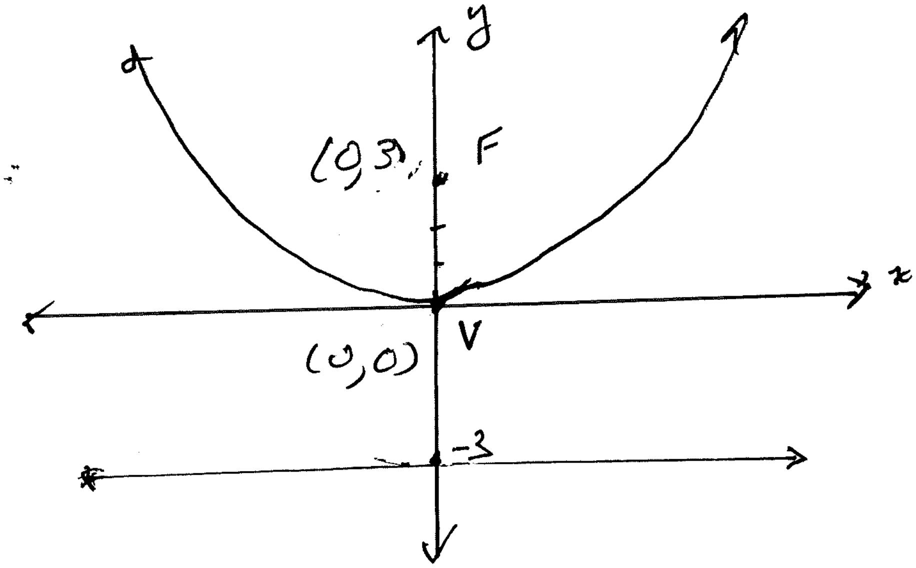
and the Directrix Equation.

Sol'n: $x^2 = 12y = 4py \Rightarrow 4p = 12 \Rightarrow p = 3 > 0$

Focus $= (0, 3)$

Vertex $= (0, 0)$

The Directrix is $y = -3$



Problem: $4y^2 - 8x - 12y + 1 = 0$

Find the Focus, Vertex, Directrix, and Sketch the Curve.

Sol'n: Vertex Shift: $(0,0) \to (h,k)$

Shifted form of the equation: $(y-k)^2 = 4p(x-h)$

Unshifted form " " : $y^2 = 4px$.

{The Focus is on the x-axis

$$4y^2 - 12y = 8x - 1 \quad (\div 4)$$

$$y^2 - 3y = 2x - \frac{1}{4} \qquad \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$y^2 - 3y + \frac{9}{4} = 2x - \frac{1}{4} + \frac{9}{4} = 2x + 2$$

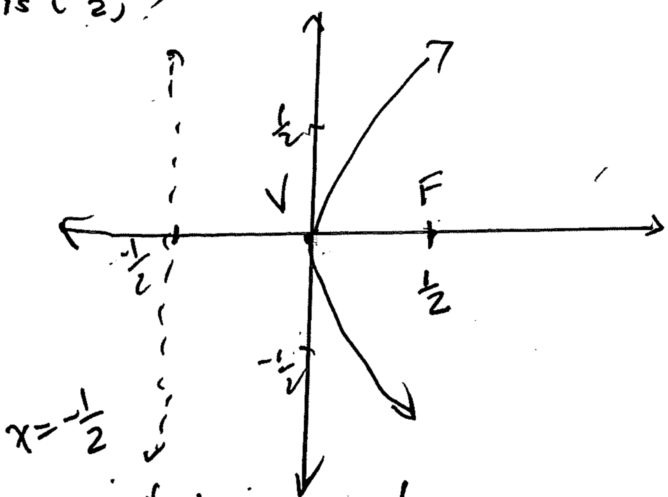
$$\left(y - \frac{3}{2}\right)^2 = 2(x+1) \qquad (h,k) = \left(-1, \frac{3}{2}\right)$$

Before Shifting $\quad (0,0) \to \left(-1, \frac{3}{2}\right)$ $\quad$ **SHIFTED**

$y^2 = 2x = 4px \Rightarrow 4p = 2$

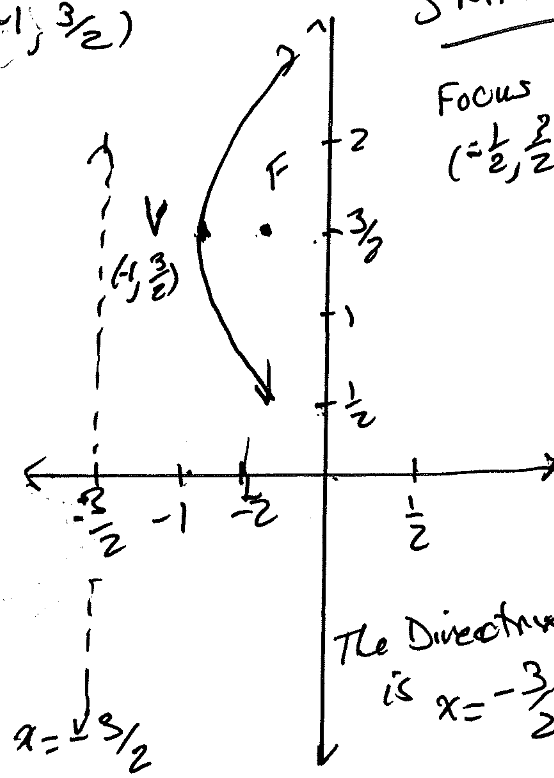
$p = \frac{1}{2}$

Focus is $\left(\frac{1}{2}, 0\right)$



The Directrix is $x = -\frac{1}{2}$

Focus $\left(-\frac{1}{2}, \frac{3}{2}\right)$



The Directrix is $x = -\frac{3}{2}$

5

1

51–54 Use a calculator to find the length of the curve correct to four decimal places. If necessary, graph the curve to determine the parameter interval.

51. One loop of the curve $r = \cos 2\theta$

52. $r = \tan\theta, \quad \pi/6 \leqslant \theta \leqslant \pi/3$

53. $r = \sin(6 \sin\theta)$

54. $r = \sin(\theta/4)$

55. (a) Use Formula 10.2.6 to show that the area of the surface generated by rotating the polar curve

$$r = f(\theta) \qquad a \leqslant \theta \leqslant b$$

(where f' is continuous and $0 \leqslant a < b \leqslant \pi$) about the polar axis is

$$S = \int_a^b 2\pi r \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

(b) Use the formula in part (a) to find the surface area generated by rotating the lemniscate $r^2 = \cos 2\theta$ about the polar axis.

56. (a) Find a formula for the area of the surface generated by rotating the polar curve $r = f(\theta)$, $a \leqslant \theta \leqslant b$ (where f' is continuous and $0 \leqslant a < b \leqslant \pi$), about the line $\theta = \pi/2$.

(b) Find the surface area generated by rotating the lemniscate $r^2 = \cos 2\theta$ about the line $\theta = \pi/2$.

10.5 Conic Sections

p. 702
(9e)

In this section we give geometric definitions of parabolas, ellipses, and hyperbolas and derive their standard equations. They are called **conic sections**, or **conics**, because they result from intersecting a cone with a plane as shown in Figure 1.

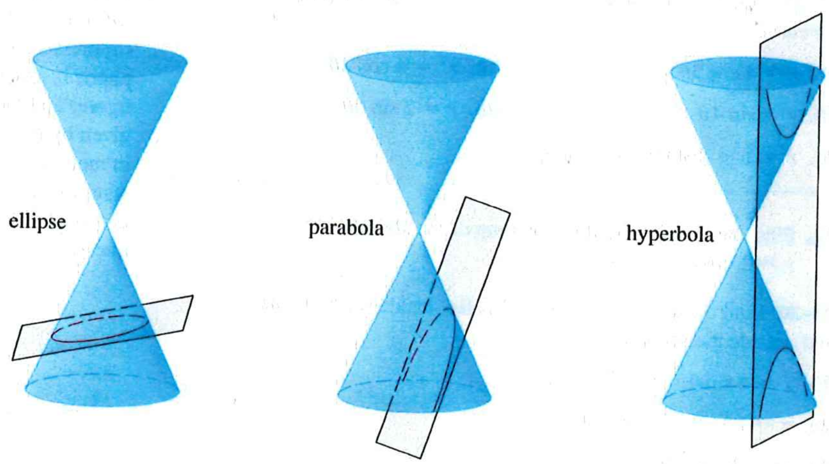


FIGURE 1
Conics

Parabolas

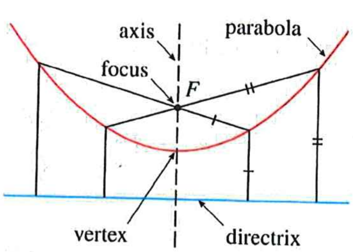


FIGURE 2

A **parabola** is the set of points in a plane that are equidistant from a fixed point F (called the **focus**) and a fixed line (called the **directrix**). This definition is illustrated by Figure 2. Notice that the point halfway between the focus and the directrix lies on the parabola; it is called the **vertex**. The line through the focus perpendicular to the directrix is called the **axis** of the parabola.

In the 16th century Galileo showed that the path of a projectile that is shot into the air at an angle to the ground is a parabola. Since then, parabolic shapes have been used in designing automobile headlights, reflecting telescopes, and suspension bridges. (See Problem 22 on page 273 for the reflection property of parabolas that makes them so useful.)

We obtain a particularly simple equation for a parabola if we place its vertex at the origin O and its directrix parallel to the x-axis as in Figure 3. If the focus is the point $(0, p)$, then the directrix has the equation $y = -p$. If $P(x, y)$ is any point on the parabola,

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Summary of the Standard Equation Forms of Conics (UNSHIFTED)

PARABOLAS:

$x^2 = 4py$, $p \neq 0$ OR $y^2 = 4px$, $p \neq 0$

THE FOCUS is $(0, p)$. THE Directrix is "$y = -p$."

THE FOCUS is $(p, 0)$. THE Directrix is "$x = -p$."

In BOTH CASES: THE VERTEX is the Origin $(0,0)$.

THE FOCUS is on the axis of the Degree 1 variable.

$|p|$ = The Distance: VERTEX TO FOCUS.

FOR ELLIPSES AND HYPERBOLAS (BOTH)

a = the DISTANCE: CENTER TO EACH VERTEX.

c = the DISTANCE: CENTER TO EACH FOCUS.

THE FOCI AND VERTICES ARE ON THE AXIS OF the Squared Variable that is over a^2.

ELLIPSES:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{OR} \quad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \text{where} \begin{cases} a > b > 0 \\ c^2 = a^2 - b^2 \\ c < a \end{cases}$$

a^2 is the LARGER DENOMINATOR.

The FOCI AND VERTICES LIE ON THE AXIS of the Variable OVER a^2.

HYPERBOLAS:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{OR} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad \text{where} \begin{cases} c^2 = a^2 + b^2 \\ c > a \end{cases}$$

a^2 is the denominator under the ADDED Squared Variable.

THE FOCI ARE ON THE AXIS OF THE ADDED Squared Variable.

The asymptotes are lines along the DIAGONALS of the "BOX"

OR